MODIFIED THERMODYNAMICS AS AN APPROACH TO THE DESCRIPTION OF SOME UNIVERSAL PROPERTIES OF "NEARLY PERFECT FLUIDS"

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We show that the quantum statistical mechanics describing quantum and thermal properties of objects has only the sense of a particular semiclassical approximation. We propose a more general (than that theory) microdescription of objects in a heat bath taking a vacuum into account as an object environment; we call it $(\hbar, k)-$dynamics ($\hbar kD$). We introduce a new generative operator, a Schrödingerian or a stochastic action operator, and will show its fundamental role in the determination of such macroquantities as internal energy, effective temperature, and effective entropy. We establish that $\hbar kD$ can serve as an initial microtheory for constructing a modified thermodynamics. On this ground, we can explain the universality of the ratio "effective action to effective entropy" at zero temperature and its minimal value in the form $\hbar/2k$. This result corresponds to experimental data obtained recently under studying a new matter state – a nearly perfect fluid.

I. EXPERIMENTAL DATA AND THE STATUS OF THE THEORY OF NEARLY PERFECT FLUIDS

In the latest years, a new direction of experimental researches has started to be developed. It has a concern with the joint analysis of the entropy density and the shear viscosity behavior in a wide temperature interval in various media: in Bose-liquids of the helium-4 type, in ultracold Bose- and Fermi-gases in traps, in traces of quark-gluon plasma at collisions of heavy nuclei, and even in graphene [1–5].

It is proved that, in so different media, the universal behavior of the ratio of the shear viscosity η to the volume density of entropy s at $T \to 0$ or $T \to T_{\rm cr}$ is observed. We note that the limiting value of the ratio is proportional to the ratio of the Planck and Boltzmann world constants. In addition, the obvious dependence of this ratio on the temperature is observed. We assume that it is not a casual result, but some evidence on the existence of a qualitatively new matter state under conditions of the strong interaction. The properties do not depend on specific features of the initial medium, but they are completely defined by the quantum-thermal influence of an environment. In the literature, this state has received the name "nearly perfect fluid" [1].

As one can see from the table below, the minimal value of the ratio η/s in units of the ratio $\hbar/k_{\rm B}$ is near to 0.5 for various media [5]:

Viscosity/entropy ratio: current status	
Quark-gluon plasma	$0 < \eta/s < 0.5$
Trapped cold alkali atoms	$(\eta/s)_{\rm min} \approx 0.5$
Liquid helium	$(\eta/s)_{\rm min} \approx 0.7$

The idea of joint consideration of the quantum and thermal stochastic influences of an environment was implicitly present at Planck's and Einstein's papers at the beginning of the XX-th century. However, till the 1970s, its realization in the well-known theories of Neumann, Matzubara, *etc.* was connected with the separate and independent account of the both types of stochastic influences. The feature of these theoretical results consists in the fact that the minimal value of entropy is equal to zero, and fluctuations of the temperature are absent. But many experiments abundantly evidence the another.

We assume that the reason for this discrepancy lies in the inconsistency of the theories. The matter is that they use a classical model of the heat bath as a set of weakly connected classical oscillators in especially quantum problems. Therefore, those theories omit the essentially quantum influence of a vacuum in the presence of the oscillator zero-point energy.

At a later time, N.N. Bogolyubov has suggested [6] (1978) to replace a classical model of the heat bath by the quantum one. This idea has opened a possibility to consider simultaneously the quantum and thermal types of stochastic influence, but it has not been used by his followers in a wide class of problems. Practically simultaneously, a close idea of thermofield vacuum has stated by Umezawa [7] who offered to describe it by a wave function with regard for the presence of a thermal noise. At the same time in the thermofield dynamics developed by him with employees, the definition of entropy corresponding to quantum statistical mechanics (QSM) was still used, and temperature fluctuations were not considered.

However, by the end of the XX-th century, some additional data have been obtained in a number of experiments which testify to an essential role of temperature fluctuations and a nonzero minimal value of entropy. These facts initiated searches of the theories giving the best consent with experiment.

In particular, in work [1], a theoretical scheme has been first offered, in which the joint account of quantum-thermal influences allowed one to get a nonzero value of entropy and a temperature-independent limiting value of the ratio η/s. On its basis, a model of the medium with "nearly perfect fluidity" as a certain collective substance was constructed. As a qualitatively essential element, the given theoretical scheme uses the relativistic quantum field theory at finite temperatures and the general relativistic theory. This model has been actively devel-

oped in publications of other authors on the basis of other approaches such as the kinetic theory, numerical modeling, and the theory of holographic duality [2, 3].

In the theory [1] developed for the model of quark-gluon plasma with strong interaction, a number of results does not correspond to experimental data for various media: first, the ratio η/s does not depend on the temperature; second, the minimal value of the ratio equals 0.08, which strongly differs from the experimental result of 0.5 quoted in the same work.

Thus, now there is a certain progress in the explanation of a limiting value of the ratio η/s for quark-gluon plasma. However, the high-grade theory suitable for an adequate explanation of the whole set of observable facts, including the universality of the given limiting ratio, is absent. Considering that these phenomena concern with the area of quantum-thermal ones, we suggest to address to the thermodynamic approach as the most universal sight at the nature. However, for our goals, the thermodynamics itself needs some modification.

The conviction that the traditional thermodynamics is completely derived from the equilibrium quantum statistical mechanics (QSM) has predominated for a quite long time. It is assumed that QSM allows establishing the observable interrelations between macroparameters (thermodynamics laws, equations of state, *etc.*) [8]. But it is well known that there exist such macroparameters (the temperature, for example), whose analogs have not yet been studied on the microlevel.

Nowadays, QSM is not able to calculate quantum and thermal phenomena simultaneously and completely. As is well known, QSM is based on the notion of the density matrix (operator). In the energy representation, it has the form of the Gibbs–von Neumann quantum canonical distribution,

$$w_n = \exp\frac{F - \varepsilon_n}{\Theta}, \tag{1}$$

where ε_n is the spectrum of the object energy, F is the free energy determined by the normalization condition, and Θ^{-1} is the modulus of the distribution.

We note the next limitations:

1. Insensitivity of Gibbs' distribution to the oscillator zero-point energy $\varepsilon_0 = \hbar\omega/2$:

$$\left.\begin{array}{l} \varepsilon_n \Rightarrow \varepsilon'_n = \varepsilon_n + \varepsilon_0 \\ F_n \Rightarrow F'_n = F_n + \varepsilon_0 \end{array}\right\} F' - \varepsilon'_n = F - \varepsilon_n. \tag{2}$$

2. Moreover, as can be seen in the ground state of the quantum oscillator, we have

$$\Delta p_0 \Delta q_0 = \frac{\hbar}{2} = \frac{\varepsilon_0}{\omega}, \tag{3}$$

which confirms the direct relation between the quantities ε_0 and $\hbar/2$. (It is principal in quantum physics!)

3. In QSM, it is assumed that the object temperature does not fluctuate. The Zero law is:

$$T = T_0; \;\; \Delta T = 0, \tag{4}$$

while the temperature fluctuations in low-temperature experiments are sufficiently noticeable for small objects, including nanoparticles and the relict radiation.

4. In addition, in QSM, Gibbs' distribution yields automatically the Third law:

$$S_{\min} = 0. \tag{5}$$

By referring to experiments, we may assert that the zeroth minimum entropy is currently very doubtful.

According to Gibbs' distribution, the expression

$$\Theta_{\rm cl} = k_{\rm B}T = \varepsilon_{\rm cl} \tag{6}$$

corresponds in QSM to choosing the classical model of the heat bath as a set of weakly coupled classical oscillators (Krylov, Bogoliubov (1939) [9]). But a microobject with quantized energy can be placed in such a heat bath even under the conditions $k_{\rm B}T < (\varepsilon_n - \varepsilon_{n-1})$. At the same time, the idea of the additivity of quantum and thermal contributions contradicts the thermal radiation theory.

II. A PROGRAMM OF CONSTRUCTING OF MODERN STOCHASTIC THERMODYNAMICS

So, as the equilibrium QSM is not a consistent theory for constructing the equilibrium thermodynamics suitable for describing the low-temperature area and small objects, we proceed from another microtheory that takes quantum-thermal effects into account, in particular, temperature fluctuations. It is the so-called ($\hbar k$)-dynamics [10]. Instead of the apparatus of density matrix, it is grounded on the complex wave function, whose amplitude and phase depend on the temperature. The suggested theory does not have the defects inherent in QSM.

On the base of this theory, we introduced a new macroparameter – the effective temperature, in which the constants $\hbar$ and $k_{\rm B}$ play equipollent roles. This temperature allows one to more fully describe the thermal equilibrium state, but it presumes using another model of heat bath. So, we start from the next statement:

Our goal is constructing a new macrotheory – the modern stochastic thermodynamics based on the evident account of the stochastic influence that is characterized on the microlevel with the stochastic action operator.

We hope that the modified thermodynamics will allow us to more closely approach the answers to the following unresolved questions:

Why does the minimal value of the ratio η/s appear universal for the state of a nearly perfect fluid in so various media?

Whether such universal ratio follows from the fundamental principles of the known standard theories – quantum mechanics and the statistical mechanics?

Whether it is necessary to use specific features of quantum field theory and general relativity theory in the description of a nearly perfect fluid in extremely various media?

III. A MODEL OF QUANTUM HEAT BATH. THE "COLD" VACUUM

In constructing the new microtheory ($\hbar kD$) as the base of the modified thermodynamics, we consider that no objects are isolated in the nature. In this connection, we follow the *Feynman approach* [11], according to which any system can be represented as a set of the object under study and its environment (the "rest of the Universe"). The environment can exert both regular and stochastic influences on the object. Here, we study only the stochastic influences. Two types of influences, namely, quantum and thermal actions characterized by the respective Planck and Boltzmann constants can be assigned to it.

So, to obtain a consistent quantum-thermal description of natural objects, we use the approach, in which we can modify the fundamental microdescription of the objects under thermal equilibrium conditions. For these purposes, we propose to formulate *a quantum-thermal dynamics* or, briefly, a ($\hbar k$)-dynamics, as a modification of the standard quantum mechanics taking thermal effects into account. The principal distinction of such a theory from QSM is that the state of a microobject in it under the conditions of contact with a quantum-heat environment is generally described not by the density matrix, but by *a temperature-dependent complex wave function.*

We note that this is not a "technical sleight-of-hand". Using the wave function, we thereby suppose to consider pure and mixed states simultaneously in the frame of *Gibbs' ensemble.* It is, in principle, differs from *Boltzmann' assembly* used in QSM.

To construct our theory, it is necessary:

1. *To change* $\hat{\rho}(T) \Rightarrow \Psi_T(q)$.
2. To introduce (except the Hamiltonian) a new operator – the *stochastic action operator* $\widehat{S}$.
3. To use an idea of heat bath at $T = 0$ (*"cold" heat bath*).
4. To use an idea of vacuum at $T > 0$ (*"thermal"* vacuum) also.

This theory is based on a new microparameter, namely, the stochastic action operator. In this case, we demonstrate that, by averaging the corresponding microparameters over the temperature-dependent wave function, we can find the *most important effective macroparameters*, including the internal energy, temperature, and entropy. They have the physical meaning of the standard thermodynamic quantities typical of a phenomenological macrodescription.

To describe the environment with the holistic stochastic influences, we introduce a specific model, the quantum heat bath (QHB) [12]. According to this, the QHB is a *set of weakly coupled quantum oscillators* with all possible frequencies. The equilibrium thermal radiation can serve as a preimage of such a model in the nature.

The specific feature of our understanding of this model is that we must apply it to both the "thermal" ($T \neq 0$) and the "cold" ($T = 0$) vacua. Thus, in the sense of Einstein [13] and Bogoliubov [6], we proceed from a more general understanding of the thermal equilibrium which can, in principle, be established for any type of environmental stochastic action (*purely quantum, quantum-thermal, and purely thermal*).

We begin our description by studying the "cold" vacuum and discussing the description of a single quantum oscillator from the number of oscillators forming the QHB model for $T = 0$ from a new standpoint.

But we recall that the lowest state in the energetic ($\Psi_n(q)$) and coherent states (CS) is the same. In the q representation, the same ground state of a quantum oscillator is, in turn, described by the *real wave function*

$$\Psi_0(q) = [2\pi(\Delta q_0)^2]^{-1/4} \exp\left\{-\frac{q^2}{4(\Delta q_0)^2}\right\}. \tag{7}$$

In the occupation number representation, the "cold" vacuum in which the number of particles is $n = 0$ corresponds to this state.

As is well known, coherent states (CS) are the eigenstates of the non-Hermitian particle annihilation operator $\hat{a}$ with complex eigenvalues. But they include one isolated state $|0_a\rangle = |\Psi_0(q)\rangle$ of the particle vacuum, in which eigenvalue of $\hat{a}$ is zero

$$\hat{a}|0_a\rangle = 0|0_a\rangle; \quad \hat{a}|\Psi_0(q)\rangle = 0|\Psi_0(q)\rangle. \tag{8}$$

In what follows, it is convenient to describe the QHB in the q, but not n representation. Therefore, we express the annihilation operator $\hat{a}$ and the creation operator $\hat{a}^\dagger$ in terms of the operators $\hat{p}$ and $\hat{q}$, using the traditional method. We have

$$\hat{a} = \frac{1}{2}\left(\frac{\hat{p}}{\sqrt{\Delta p_0^2}} - i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}\right);$$

$$\hat{a}^\dagger = \frac{1}{2}\left(\frac{\hat{p}}{\sqrt{\Delta p_0^2}} + i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}\right), \tag{9}$$

where $(\Delta q_0)^2 = \hbar/2m\omega$ and $(\Delta p_0)^2 = \hbar m\omega/2$

The particle number operator then becomes

$$\hat{N}_a = \hat{a}^\dagger\hat{a} = \frac{1}{\hbar\omega}\left(\frac{\hat{p}^2}{2m} - \frac{\hbar\omega}{2}\hat{I} + \frac{m\omega^2\hat{q}^2}{2}\right). \tag{10}$$

After multiplying this relation by $\hbar\omega$, we obtain the standard interrelation between the expressions for the Hamiltonian in the q and n representations:

$$\hat{\mathcal{H}} = \frac{\hat{p}^{\,2}}{2m} + \frac{m\omega^2\hat{q}^2}{2} = \hbar\omega(\hat{N}_a + \frac{1}{2}\hat{I}), \tag{11}$$

where $\hat{I}$ is the unit operator.

From the thermodynamics standpoint, we are concerned with the internal energy of the quantum oscillator in equilibrium with the "cold" QHB. Its value is equal to

$$U_0 = \langle\Psi_0(q)|\hat{\mathcal{H}}|\Psi_0(q)\rangle =$$

$$= \hbar\omega\langle\Psi_0(q)|\hat{N}_a|\Psi_0(q)\rangle + \frac{\hbar\omega}{2} = \frac{\hbar\omega}{2} = \varepsilon_0. \tag{12}$$

In the given case, the state without particles coincides with the state of the Hamiltonian with the minimum energy ε_0. *The quantity* ε_0 traditionally treated as the oscillator zero-point energy takes the physical meaning *of the internal energy* U_0 of a quantum oscillator in equilibrium with the "cold" vacuum.

IV. A MODEL OF QUANTUM HEAT BATH. THE "THERMAL" VACUUM

We can pass from the "cold" to the "thermal" vacuum using the Bogoliubov (u, v)-transformation with the *temperature-dependent coefficients* [12]

$$u = \left(\frac{1}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T} + \frac{1}{2}\right)^{1/2} e^{i\frac{\pi}{4}};$$

$$v = \left(\frac{1}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T} - \frac{1}{2}\right)^{1/2} e^{-i\frac{\pi}{4}}. \tag{13}$$

In the given case, this transformation is canonical but leads to a unitary nonequivalent representation, because the QHB at any temperature is a system with an infinitely large number of degrees of freedom. Such a transformation allows passing from the set of CS to a more general set of states called the *thermal correlated coherent states*. They are selected because they ensure that the Schrödinger coordinate - momentum uncertainties relation for a quantum oscillator is saturated at any temperature.

From the second-quantization apparatus standpoint, the transformation ensures the passage from the original system of *particles* with the "cold" vacuum $|0_a\rangle$ to the system of *quasiparticles* described by the annihilation operator $\hat{b}$ and the creation operator $\hat{b}^\dagger$ with the "thermal" vacuum $|0_b\rangle$.

So, we pass

1. From cold to thermal vacuum:

$$\Psi_0(q) \Rightarrow \Psi_T(q); \quad |0_a\rangle \Rightarrow |0_b\rangle.$$

2. From particles to quasiparticles:

$$\hat{a} \Rightarrow \hat{b} = \hat{b}(T); \quad \hat{a}^\dagger \Rightarrow \hat{b}^\dagger = \hat{b}^\dagger(T).$$

The choice of transformation coefficients is fixed by the requirement that the expression for the mean energy of a quantum oscillator in the thermal equilibrium be defined by the Planck formula, which can be obtained from experiments:

$$\varepsilon_{\mathrm{Pl}} = \hbar\omega(\exp\frac{\hbar\omega}{k_{\mathrm{B}}T} - 1)^{-1} + \frac{\hbar\omega}{2} = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T} = \varepsilon_{qu}. \tag{14}$$

Earlier, it was shown by us [12] that the state of the *"thermal" vacuum* $|0_b\rangle \equiv |\Psi_T(q)\rangle$ in the q representation corresponds to the complex wave function

$$\Psi_T(q) = [2\pi(\Delta q)^2]^{-1/4} \exp\left\{-\frac{q^2}{4(\Delta q)^2}(1 - i\alpha)\right\}, \tag{15}$$

where

$$(\Delta q)^2 = \frac{\hbar}{2m\omega}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}; \quad \alpha = \left[\sinh\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right]^{-1}. \tag{16}$$

For its Fourier transform $\Psi_T(p)$, a similar expression with the same coefficient α and $(\Delta p)^2 = \frac{\hbar m\omega}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}$ holds.

V. SOME FEATURES OF THE "THERMAL" VACUUM

Of course, the states from the set of thermal correlated coherent states are the eigenstates of the non-Hermitian quasiparticle annihilation operator $\hat{b}$ with complex eigenvalues. They also include one isolated state of the *quasiparticle vacuum*, in which eigenvalue of $\hat{b}$ is zero,

$$\hat{b}|0_b\rangle = 0|0_b\rangle; \quad \hat{b}|\Psi_T(q)\rangle = 0|\Psi_T(q)\rangle. \tag{17}$$

Using the wave function of the "thermal" vacuum, we obtain the expression for the operator $\hat{b}$ in the q representation:

$$\hat{b} = \frac{1}{2}\sqrt{\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})}\times$$

$$\times\left[\frac{\hat{p}}{\sqrt{\Delta p_0^2}} - i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})^{-1}(1 - i\alpha)\right]. \tag{18}$$

The corresponding quasiparticle creation operator $\hat{b}^\dagger$ can be obtained from $\hat{b}$ with i replaced by $(-i)$

$$\hat{b}^\dagger = \frac{1}{2}\sqrt{\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})}\times$$

$$\times\left[\frac{\hat{p}}{\sqrt{\Delta p_0^2}} + i\frac{\hat{q}}{\sqrt{\Delta q_0^2}}(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})^{-1}(1 + i\alpha)\right]. \tag{19}$$

We can verify that, as $T \to 0$, the operators $\hat{b}^\dagger$ and $\hat{b}$ for quasiparticles pass to the operators $a^\dagger$ and $\hat{a}$ for particles, and

$$|0_b\rangle \Rightarrow |0_a\rangle; \quad \Psi_T(q) \Rightarrow \Psi_0(q).$$

Acting just as above, we obtain the expression for the *quasiparticle number operator* $\hat{N}_b$ in the q representation

$$\hat{N}_b = \hat{b}^\dagger\hat{b} = \frac{1}{4}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T}\times$$

$$\times\left[\frac{\hat{p}^2}{\Delta p_0^2} - 2(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})^{-1}(\hat{I} + \frac{\alpha}{\hbar}\{\hat{p}, \hat{q}\}) + \frac{\hat{q}^2}{\Delta q_0^2}\right], \tag{20}$$

where we take $1 + \alpha^2 = \coth^2\frac{\hbar\omega}{2k_{\mathrm{B}}T}$ into account when calculating the last term.

Multiplying by $\hbar\omega$ and passing from the quasiparticle number operator to the original Hamiltonian, we obtain

$$\hat{\mathcal{H}} = \hbar\omega(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})^{-1}\left[\hat{N}_b + \frac{1}{2}\hat{I} + \frac{\alpha}{\hbar}\{\hat{p}, \hat{q}\}\right]. \tag{21}$$

We stress that the operator $\{\hat{p}, \hat{q}\}$ in this formula can also be expressed in terms of bilinear combinations of the operators $\hat{b}^{\dagger}$ and $\hat{b}$, but they differ from the quasiparticle number operator. This means that the operators $\hat{\mathcal{H}}$ and $\hat{N}_b$ do *not commute*, and the wave function $\psi_T(q)$ is therefore *not the eigenfunction of the Hamiltonian* $\hat{\mathcal{H}}$.

As before, we are interested in the thermodynamic quantity, namely, the internal energy U of a quantum oscillator now in thermal equilibrium with the "thermal" QHB. Calculating it just as earlier, we obtain

$$U = \hbar\omega(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})^{-1}\times$$

$$\times\left[\langle\Psi_T(q)|\hat{N}_b|\Psi_T(q)\rangle + \frac{1}{2} + \frac{\alpha}{2\hbar}\langle\Psi_T(q)|\{\hat{p}, \hat{q}\}|\Psi_T(q)\rangle\right] \quad (22)$$

in the q representation.

Because we average over the quasiparticle vacuum in this formula, the first term in it vanishes. At the same time, it was shown earlier by us [14] that

$$\langle\Psi_T(q)|\{\hat{p}, \hat{q}\}|\Psi_T(q)\rangle = \hbar\alpha. \quad (23)$$

As a result, we obtain the expression for the internal energy of the quantum oscillator in the "thermal" QHB in the $\hbar kD$:

$$U = \frac{\hbar\omega}{2(\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T})}(1+\alpha^2) = \frac{\hbar\omega}{2}\coth\frac{\hbar\omega}{2k_{\mathrm{B}}T} = \varepsilon_{\mathrm{Pl}}. \quad (24)$$

This means that the average energy of the quantum oscillator at $T \neq 0$ has the thermodynamic meaning of its internal energy in the case of equilibrium with the "thermal" vacuum. As $T \to 0$, it passes to a similar quantity corresponding to the equilibrium with the "cold" vacuum.

VI. THE STOCHASTIC ACTION OPERATOR – SCHRÖDINGERIAN

Because the original statement of the $\hbar kD$ is the *idea of the holistic stochastic action* of the QHB on the object, we want to introduce a new operator in the Hilbert space of microobject states to implement it.

In this connection, we recall the Schwartz inequality

$$|A|^2 \cdot |B|^2 \geq |A \cdot B|^2, \quad (25)$$

the Schrödinger uncertainty relation (SUR) for the coordinate and the momentum following from it [15]:

1. Unsaturated SUR

$$(\Delta p)^2(\Delta q)^2 > |\tilde{R}_{qp}|^2 \equiv \sigma^2 + \frac{\hbar^2}{4}. \quad (26)$$

2. Saturated SUR

$$(\Delta p)^2(\Delta q)^2 = |\tilde{R}_{qp}|^2. \quad (27)$$

In the absent of a stochastic action, $\tilde{R}_{qp} \equiv 0$. For real states, $\sigma = 0$.

Hereinafter, we use an analysis of the right-hand side of the saturated coordinate-momentum SUR. For not only a quantum oscillator in a heat bath but also for any object, the complex quantity on the right-hand side of the SUR,

$$\widetilde{R}_{p\,q} = \langle\Delta p|\Delta q\rangle \quad \text{or} \quad \widetilde{R}_{p\,q} = \langle\,|\Delta\widehat{p}\,\Delta\widehat{q}\,|\,\rangle, \quad (28)$$

has a double meaning.

On the one hand, it is the *amplitude of the transition* from the state $|\Delta q\rangle$ to the state $|\Delta p\rangle$. On the other hand, it can be treated as the *quantum correlator* calculated over an arbitrary state $|\ \rangle$ of some operator. We note that it has a dimension of action.

The *nonzero value* of the quantity $\widetilde{R}_{pq}$ is the *fundamental attribute of nonclassical theory.* Therefore, it is quite natural to assume that the averaged operator in that formula has a fundamental meaning. In view of dimensional considerations, we call it **the stochastic action operator or Schrödingerian**,

$$\widehat{S} \equiv \Delta\widehat{p}\,\Delta\widehat{q}. \quad (29)$$

At the first time, it was introduced in the paper by E. Schrödinger (1930) [16]. Of course, it should be remembered that the operators $\Delta\hat{q}$ and $\Delta\hat{p}$ do not commute, and their product is a non-Hermitian operator.

We can express the given operator in the form

$$\widehat{S} = \frac{1}{2}\{\Delta\widehat{p}\Delta\widehat{q} + \Delta\widehat{q}\Delta\widehat{p}\} + \frac{1}{2}[\Delta\widehat{p}\Delta\widehat{q} - \Delta\widehat{q}\Delta\widehat{p}] = \widehat{\sigma} - i\,\widehat{S}_0. \quad (30)$$

It allows separating the Hermitian part of $\widehat{S}$ from the anti-Hermitian one. Then the Hermitian operators $\widehat{\sigma}$ and $\widehat{S}_0$ have the form

$$\widehat{\sigma} \equiv \frac{1}{2}\{\Delta\widehat{p},\ \Delta\widehat{q}\}; \quad \widehat{S}_0 \equiv \frac{i}{2}\,[\widehat{p}, \widehat{q}] = \frac{\hbar}{2}\,\hat{I}. \quad (31)$$

It is easy to see that the mean $\sigma = \langle\,|\widehat{\sigma}|\,\rangle$ of the operator $\hat{\sigma}$ resembles the expression for the standard correlator of coordinate and momentum fluctuations in classical probability theory. It transforms into this expression if the operators $\Delta\hat{q}$ and $\Delta\hat{p}$ are replaced with c-numbers. It reflects the contribution of the environmental stochastic action to the transition amplitude $\tilde{R}_{pq}$.

Therefore, we call the operator $\hat{\sigma}$ *the external action operator* in what follows. Previously, the possibility to use a similar operator $\hat{\sigma}$ was discussed by Krylov and Bogoliubov [9], where it was studied as a *quantum analog* of the action variable in the set of action-angle classical variables.

At the same time, the operators $\widehat{S}_0$ and $\widehat{S}$ were *not previously introduced.* The operator $\widehat{S}_0$ reflects a specific peculiarity of the objects to be "sensitive" to the minimum stochastic action of the "cold" vacuum and to respond to it adequately regardless of their states.

Therefore, it should be treated as a *minimal stochastic action operator.* Its mean

$$\langle\,|\hat{S}_0|\,\rangle \equiv J_0 = \frac{\hbar}{2} \quad (32)$$

is independent of the choice of the state, over which the averaging is performed, and it has, hence, the meaning of the invariant eigenvalue of the operator $\widehat{S}_0$.

This implies that, in the given case, we deal with the universal quantity J_0 which is called the minimal action. Its fundamental character is already defined by its relation to the Planck world constant $\hbar$. But the problem is not settled yet.

Indeed, according to the tradition dating back to Planck, the quantity $\hbar$ is assumed to be called the elementary quantum of action. At the same time, the factor 1/2 in the quantity J_0 plays a significant role, while half the quantum of the action is not observed in the nature.

Therefore, the quantities $\hbar$ and $\frac{1}{2}\hbar$, whose dimensions coincide, have different physical meanings and must be named differently, in our opinion. From this standpoint, it would be more natural to call the quantity $\hbar$ the external action quantum. Hence, the quantity $\hbar$ is the minimum portion of the action transferred to the object from the environment or from another object. Therefore, photons and other quanta of fields being carriers of fundamental interactions are first the carriers of the minimal action equal to $\hbar$. The same is also certainly related to phonons.

Finally, we note that only the quantity $\hbar$ is related to the discreteness of the spectrum of the quantum oscillator energy in the absence of the heat bath. At the same time, the quantity $\frac{\hbar}{2}$ has an independent physical meaning. It specifies the minimum value of the macroparameter – the internal energy U_0 of the quantum oscillator in the "cold" QHB (at $T = 0$).

Below, we wil evaluate the specific features of the Schrödingerian used in the microdescription. We recall that this operator is non-Hermitian. This fact would seemingly contradict the standard requirements imposed on the operators in quantum mechanics, but there is nothing unusual in this.

If we are interested in genuine quantum dynamics which is naturally associated with transitions from one state to another one, then precisely the non-Hermitian operators play an important role. The creation and annihilation operators or, for example, the scattering matrix are among the most well-known of them. The stochastic action operator $\widehat{S}$ also belongs to these operators.

VII. EFFECTIVE ACTION AS A FUNDAMENTAL MACROPARAMETER

We now construct the macrodescription of objects using their microdescription in the $\hbar kD$. The mean $\widetilde{S}$ of the operator $\widehat{S}$ coincides with the complex transition amplitude $\widetilde{R}_{pq}$ or quantum correlator and, in thermal equilibrium, can be expressed as

$$\widetilde{S} = \langle\Psi_T(q)|\,\widehat{S}\,|\Psi_T(q)\rangle = \sigma -\, i\,J_0, \tag{33}$$

where σ and J_0 are the means of the corresponding operators. In what follows, we regard the modulus of the complex quantity $\widetilde{S}$,

$$|\widetilde{S}| = \sqrt{\sigma^2 + J_0^2} = \sqrt{\sigma^2 + \frac{\hbar^2}{4}} \equiv J_{\rm ef} \tag{34}$$

as a new macroparameter – the effective action. For the quantum oscillator, it has the form

$$J_{\rm ef} = \frac{\hbar}{2}\coth\frac{\hbar\omega}{2k_{\rm B}T} \tag{35}$$

and coincides with a quantity previously postulated from intuitive considerations [12].

Now we establish the interrelation between the effective action and traditional thermodynamic quantities. Comparing the expressions for $J_{\rm ef}$ and U, we can see that $U = \omega|\widetilde{S}| = \omega J_{\rm ef}$. In the high-temperature limit, where $\sigma \to \sigma_T = k_{\rm B}T/\omega \gg \hbar/2$, this relation becomes $U = \omega\sigma_T$. This formula was obtained by Boltzmann [17] (1904) for macroparameters in classical thermodynamics by generalizing the concept of adiabatic invariants in classical mechanics.

The formula obtained above also allows expressing the interrelation between the effective action $J_{\rm ef}$ and the effective temperature $T_{\rm ef}$ [18] in the explicit form:

$$T_{\rm ef} = \frac{\varepsilon_{qu}}{k_{\rm B}} = \frac{\hbar\omega}{2k_{\rm B}}\coth\frac{\hbar\omega}{2k_{\rm B}T} = \frac{\omega}{k_{\rm B}}J_{\rm ef}. \tag{36}$$

This yields

$$T_{\rm ef}^{min.} = \frac{\omega}{k_{\rm B}}S_0 = \frac{\hbar\omega}{2k_{\rm B}} \neq 0. \tag{37}$$

VIII. EFFECTIVE ENTROPY IN THE $\hbar kD$

The possibility of introducing the entropy in the $\hbar kD$ is also based on using the wave function instead of the density operator.

Using the corresponding dimensionless expressions $\tilde\rho(\tilde q)$ and $\tilde\rho(\tilde p)$ instead of $\rho(q) = |\Psi_T(q)|^2$ and $\rho(p) = |\Psi_T(p)|^2$, we propose to define a formal coordinate – momentum entropy S_{qp} by the equality

$$S_{qp} = -k_{\rm B}\left\{\int\tilde\rho(\tilde q)\ln\tilde\rho(\tilde q)d\tilde q + \int\tilde\rho(\tilde p)\ln\tilde\rho(\tilde p)d\tilde p\right\}. \tag{38}$$

Substituting the corresponding dimensionless expressions and using the normalizing condition, we obtain

$$S_{qp} = k_{\rm B}\left\{(1 + \ln\frac{2\pi}{\delta}) + \ln\coth\frac{\hbar\omega}{2k_{\rm B}T}\right\}. \tag{39}$$

The final result depends on the choice of the constant δ. If $\delta = 2\pi$, we can interpret the given expression as the quantum-thermal entropy or, briefly, S_{QT}, because it coincides exactly with

$$S_{QT} \equiv S_{\rm ef} = k_{\rm B}\left\{1 + \ln\coth\frac{\hbar\omega}{2k_{\rm B}T}\right\} \to k_{\rm B}, \tag{40}$$

obtained earlier by us in the macrotheory framework [12].

This expression, as distinct from the entropy in QSM, takes the averaging both of the amplitude $\overline{|\Psi_T(q)|}$ and the phase $\overline{\varphi} = \frac{\alpha}{4}$ into account.

IX. STANDARD THERMODYNAMICS ON THE BASE OF THE EFFECTIVE ACTION

Using the $\hbar kD$, we can introduce the effective action $J_{\rm ef}$ as a new fundamental macroparameter. Its advantage is that it has a *microscopic preimage* – the stochastic action operator $\widehat{S}$ or Schrödingerian. Moreover, in thermal equilibrium, we can express the main thermodynamic characteristics of objects – temperature and entropy – in terms of it.

For example, we recall that $T_{\rm ef} \sim J_{\rm ef}$. It follows that $J_{\rm ef}$ is also an intensive macroparameter characterizing the stochastic action of the "thermal" QHB.

In view of this, the *Zero law* of equilibrium modern stochastic thermodynamics instead of

$$T_{\rm ef} = T_{\rm ef}^{\rm therm} \pm \Delta T_{\rm ef}, \tag{41}$$

can be rewritten as

$$J_{\rm ef} = J_{\rm ef}^{\rm therm} \pm \Delta J_{\rm ef}, \tag{42}$$

where $J_{\rm ef}^{\rm therm}$ is effective action of QHB, and $J_{\rm ef}$ and $\Delta J_{\rm ef}$ are, respectively, the means of the effective action of an object and the standard deviation from it. The state of thermal equilibrium can actually be described, in such cases, in the sense of Newton, assuming that

"the stochastic action is equal to the stochastic counteraction".

We now turn to the effective entropy. In the absence of a mechanical contact,

$$dS_{\rm ef} = \frac{\delta Q_{\rm ef}}{T_{\rm ef}} = \frac{dU}{T_{\rm ef}}. \tag{43}$$

Now we can rewrite this expression in the form

$$dS_{\rm ef} = k_{\rm B} d(\ln \frac{J_{\rm ef}}{S_0}) \equiv dS_{QT}. \tag{44}$$

It follows that the effective or QT-entropy, being an extensive macroparameter, can be also expressed in terms of $J_{\rm ef}$. Correspondingly, *the third law* is in accordance with Nernst initial statement

$$S_{QT} = S_{\rm ef} = k_{\rm B}\{1 + \ln\coth \frac{J_{\rm ef}}{J_0}\} \to k_{\rm B} \neq 0. \tag{45}$$

X. SUMMARY

Nowadays, there is a number of experiments and theories where the ratio η/s is the subject of interest. It was shown by us [12] that this quantity can be given by

$$\frac{\eta}{s} \equiv \frac{J^{\rm ef}}{S^{\rm ef}} = \xi \, \frac{\coth \xi\omega/T}{1 + \ln\coth \xi\omega/T} \to \xi, \tag{46}$$

where

$$\xi \equiv \frac{J^{\rm ef}_{\rm min}}{S^{\rm ef}_{\rm min}} = \frac{\hbar}{2k_{\rm B}} = 3.8 \times 10^{-12} \ \text{K}\cdot \text{c} \tag{47}$$

and $J^{\rm ef}_{\rm min} = J_0 \equiv \hbar/2$, $S^{\rm ef}_{\rm min} = k_{\rm B}$ are the limiting values for $T \ll T^{\rm ef}$.

In contrast to the $\hbar kD$, QSM yields

$$\frac{\eta}{s} \to \frac{\hbar \exp(-\hbar\omega/k_{\rm B}T)}{k_{\rm B}(\hbar\omega/k_{\rm B}T)\exp(-\hbar\omega/k_{\rm B}T)} = \frac{T}{\omega} \to 0. \tag{48}$$

So, it is possible to compare two theories ($\hbar kD$ and QSM) experimentally by measuring the limiting value of this ratio: the ratio $\frac{\eta}{s}$ is equal to either ξ or zero.

So, we have proposed the $(\hbar k)$-dynamics as a more general microdescription of objects in a heat bath with the vacuum explicitly taken into account than that presented by QSM. We have constructed a basically new model of the object environment, namely, a quantum heat bath. We have studied its properties including the cases of "cold" and "thermal" vacua.

We have introduced a new stochastic action operator – Schrödingerian – and shown its fundamental role in the microdescription. We have established that the corresponding macroparameter, the effective action $J_{\rm ef}$, plays just a significant role in the macrodescription. The most important effective macroparameters of equilibrium quantum statistical thermodynamics such as the internal energy, temperature, and entropy are expressed in terms of this macroparameter.

So, we have found, in part, the answer to the questions: Why is this ratio universal for different nearly perfect fluids? Does this ratio follow from the general principles of quantum mechanics and statistical mechanics?

We suppose that, yes. The ratio is an universal one, and it follows from the general principles of not quantum mechanics or statistical mechanics taken separately, but of a more generalized joint theory – the $(\hbar, k)$-dynamics.

As a result, it appears that the stochastization process of object characteristics due to the contact with QHB is described at the macrolevel by two qualitatively various quantities – effective entropy and effective temperature. Nevertheless, at any absolute temperatures, they can be expressed through the same macroparameter – effective influence. It is important to notice that, in the $(\hbar, k)$-dynamics, the last value has a microscopic prototype. It is the stochastic influence operator or Schrödingerian depending on the world constants of Planck and Boltzmann.

So, it is necessary to stress that the theory offered above, the $(\hbar, k)$-dynamics, not only has already allowed explaining the universality of the limiting value of the ratio $\frac{\eta}{s}$ observed in experiments. It has established a microscopic prototype of such a quantity as the temperature. As we have pointed earlier, QSM is not able to do it, because there is not a suitable operator like the Schrödingerian in it. At its further generalization to nonequilibrium wave functions, the given theory can also play the important role in the explanation of various properties of nearly perfect fluids.

However, the area of its applicability can be wider. The fundamental idea of the transition from a classical model

of the heat bath to a quantum model automatically leads to the necessity of the replacement

$$k_{\mathrm{B}}T \to k_{\mathrm{B}}T_{ef}$$

in the majority of the formulas deduced earlier within the limits of thermodynamics and statistical mechanics.

Doing this replacement, we open a possibility of a deeper analysis of the existing experiments under conditions of equilibrium with QHB at ultralow temperatures. In this case, some quantum effects can be revealed, by obviously reflecting the presence of quantum stochastic influences of the "cold" vacuum. Certainly, the discovery of such effects will allow interpreting, in a new manner, many already known quantum effects shown at ultralow temperatures.

This work was supported by the Russian Foundation for Basic Research (Project No. 10-01-90408).

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Received 28.09.10